Exercise 3B

1 a i Yes, since n = 120 is large (> 50) and p = 0.6 is close to 0.5.

ii $\mu = np = 120 \times 0.6 = 72$ and $\sigma = \sqrt{np(1-p)} = \sqrt{72 \times 0.4} = \sqrt{28.8} = 5.37$ (3 s.f.)
$X \sim \mathrm{B}(72, 5.37^2)$

b i No, n = 20 is not large enough (< 50).

c i Yes, since n = 250 is large (> 50) and p = 0.52 is close to 0.5.

ii $\mu = np = 250 \times 0.52 = 130$ and $\sigma = \sqrt{np(1-p)} = \sqrt{130 \times 0.48} = \sqrt{62.4} = 7.90$ (3 s.f.)
$X \sim \mathrm{B}(130, 7.90^2)$

d i No, p = 0.85 is too far from 0.5.

e i Yes, since n = 400 is large (> 50) and p = 0.48 is close to 0.5.

ii $\mu = np = 400 \times 0.48 = 192$ and $\sigma = \sqrt{np(1-p)} = \sqrt{192 \times 0.52} = \sqrt{99.84} = 9.99$ (3 s.f.)
$X \sim \mathrm{B}(192, 9.99^2)$

f i Yes, since n = 1000 is large (> 50) and p = 0.58 is close to 0.5.

ii $\mu = np = 1000 \times 0.58 = 580$ and $\sigma = \sqrt{np(1-p)} = \sqrt{580 \times 0.42} = \sqrt{243.6} = 15.6$ (3 s.f.)
$X \sim \mathrm{B}(580, 15.6^2)$

2 A normal approximation is valid since n = 150 is large (> 50) and p = 0.45 is close to 0.5.
$\mu = np = 150 \times 0.45 = 67.5$ and $\sigma = \sqrt{np(1-p)} = \sqrt{67.5 \times 0.55} = \sqrt{37.125} = 6.093$ (4 s.f.)
So X can be approximated by $Y \sim \mathrm{N}(67.5, 6.093^2)$

a $\mathrm{P}(X \leqslant 60) \approx \mathrm{P}(Y < 60.5) = 0.1253$ (4 d.p.)

b $\mathrm{P}(X > 75) \approx \mathrm{P}(Y > 75.5) = 0.0946$ (4 d.p.)

c $\mathrm{P}(65 \leqslant X \leqslant 80) \approx \mathrm{P}(64.5 < Y < 80.5) = 0.6723$ (4 d.p.)

3 A normal approximation is valid since n = 200 is large (> 50) and p = 0.53 is close to 0.5.
$\mu = np = 200 \times 0.53 = 106$ and $\sigma = \sqrt{np(1-p)} = \sqrt{106 \times 0.47} = \sqrt{49.82} = 7.058$ (4 s.f.)
So X can be approximated by $Y \sim \mathrm{N}(106, 7.058^2)$

a $\mathrm{P}(X < 90) \approx \mathrm{P}(Y < 89.5) = 0.0097$ (4 d.p.)

b $\mathrm{P}(100 \leqslant X < 110) \approx \mathrm{P}(99.5 < Y < 109.5) = 0.5115$ (4 d.p.)

c $\mathrm{P}(X = 105) \approx \mathrm{P}(104.5 < Y < 105.5) = 0.0559$ (4 d.p.)

4 A normal approximation is valid since $n = 100$ is large (> 50) and $p = 0.6$ is close to 0.5.
$\mu = np = 100 \times 0.6 = 60$ and $\sigma = \sqrt{np(1-p)} = \sqrt{60 \times 0.4} = \sqrt{24} = 4.899$ (4 s.f.)
So X can be approximated by $Y \sim \mathrm{N}(60, 4.899^2)$

a $\mathrm{P}(X > 58) \approx \mathrm{P}(Y > 58.5) = 0.6203$ (4 d.p.)

b $\mathrm{P}(60 < X \leqslant 72) \approx \mathrm{P}(60.5 < Y < 72.5) = 0.4540$ (4 d.p.)

c $\mathrm{P}(X = 70) \approx \mathrm{P}(69.5 < Y < 70.5) = 0.0102$ (4 d.p.)

5 Let X be the number of heads in 70 tosses of a fair coin, so $X \sim \mathrm{B}(70, 0.5)$.
Since $p = 0.5$ and 70 is large, X can be approximated by the normal distribution $Y \sim \mathrm{N}(\mu, \sigma^2)$,
where $\mu = 70 \times 0.5 = 35$ and $\sigma = \sqrt{70 \times 0.5 \times 0.5} = \sqrt{17.5}$
So $Y \sim \mathrm{N}(35, 17.5)$
$\mathrm{P}(X > 45) \approx \mathrm{P}(Y \geqslant 45.5) = 0.0060$ (4 d.p.)

6 A normal approximation is valid since $n = 1200$ is large and p is close to 0.5.
$$\mu = np = 1200 \times \frac{50}{101} = 594.059$$
and $\sigma = \sqrt{np(1-p)} = \sqrt{594.059 \times \frac{51}{101}} = \sqrt{299.97059\ldots} = 17.32$ (4 s.f.)
So $Y \sim \mathrm{N}(594.059, 299.97...)$
$\mathrm{P}(X \geqslant 600) \approx \mathrm{P}(Y > 599.5) = 0.3767$ (4 d.p.)

7 **a** The number of trials, n, must be large (> 50), and the success probability, p, must be close to 0.5.

b Using the binomial distribution, $\mathrm{P}(X = 10) = \binom{20}{10} \times 0.45^{10} \times 0.55^{10} = 0.1593$ (4 d.p.)

c A normal approximation is valid since $n = 240$ is large and $p = 0.45$ is close to 0.5.
$\mu = np = 240 \times 0.45 = 108$ and $\sigma = \sqrt{np(1-p)} = \sqrt{108 \times 0.55} = \sqrt{59.4} = 7.707$ (4 s.f.)
So $Y \sim \mathrm{N}(108, 59.4)$
$\mathrm{P}(X < 110) \approx \mathrm{P}(Y < 109.5) = 0.5772$ (4 d.p.)

d $\mathrm{P}(X \geqslant q) = 0.2 \Rightarrow \mathrm{P}(Y > (q - 0.5)) = 0.2$
Using the inverse normal function,
$\mathrm{P}(Y > (q - 0.5)) = 0.2 \Rightarrow q - 0.5 = 114.485 \Rightarrow q = 114.985$
So $q = 115$

8 **a** Using the cumulative binomial function with $N = 30$ and $p = 0.52$,
$\mathrm{P}(X < 17) = \mathrm{P}(X \leqslant 16) = 0.6277$ (4 d.p.)

b A normal approximation is valid since $n = 600$ is large and $p = 0.52$ is close to 0.5.
$\mu = np = 600 \times 0.52 = 312$ and $\sigma = \sqrt{np(1-p)} = \sqrt{312 \times 0.48} = \sqrt{149.76} = 12.24$ (4 s.f.)
So $Y \sim \mathrm{N}(312, 149.76)$
$\mathrm{P}(300 \leqslant X \leqslant 350) \approx \mathrm{P}(299.5 < Y < 350.5) = 0.8456$ (4 d.p.)

9 a Using the binomial distribution, $P(X=55)=\binom{100}{55}\times 0.56^{55}\times 0.44^{45}=0.0784$ (4 d.p.)

b A normal approximation is valid since $n = 100$ is large and $p = 0.56$ is close to 0.5.
$\mu = np = 100\times 0.56 = 56$ and $\sigma=\sqrt{np(1-p)}=\sqrt{56\times 0.44}=\sqrt{24.64}=4.964$ (4 s.f.)
So $Y \sim B(56, 24.64)$
$P(X=55)\approx P(54.5<Y<55.5)=0.07863$ (4 s.f.)
Percentage error = $\frac{0.07838...(-)0.07863...}{0.07838...}\times 100 = 0.31\%$ (2 d.p.)